Physics Solution Paper - 01
SECTION - I
Single Correct Choice Type

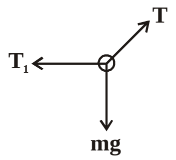
1. (a) $1\sin 60° = \mu_g \sin 35°$; $1\sin 60° = \mu_w \sin 47°$

$\mu_w \sin 45° = \mu_g \sin\theta$;

$\mu_w \sin 47° = \mu_g \sin 35°$

$\frac{\sin 45°}{\sin 47°} \times \sin 35 = \sin\theta \Rightarrow \theta < 35$

2. (a)



$T\cos 45° = mg$, ; $T_1 = T\sin 45°$,

$T_1 = mg$.

3. (c) From given curve,

Melting point for $A = 60°C$

and melting point for $B = 20°C$

Time taken by *A* for fusion $= (6-2) = 4$ minute

Time taken by *B* for fusion $= (6.5-4) = 2.5$ minute

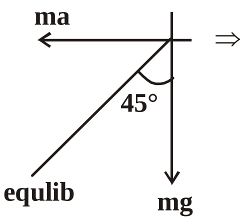
Then $\frac{H_A}{H_B} = \frac{6\times 4\times 60}{6\times 2.5\times 60} = \frac{8}{5}$.

4. (c) When loop is entering in the field, magnetic flux (*i.e.* ×) linked with the loop increases so induced emf in it $e = Bvl = 0.6\times 10^{-2}\times 5\times 10^{-2} = 3\times 10^{-4}V$ (Negative)

When loop completely entered in the field (after 5 *sec*) flux linked with the loop remains constant so $e = 0$.

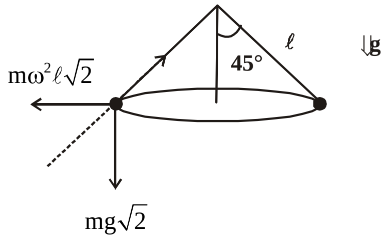
After 15 *sec*, loop begins to exit out, linked magnetic flux decreases so induced emf $e = 3\times 10^{-4}V$ (Positive).

5. (c)



$\Rightarrow$ $a = g$, $g_{eff} = g\sqrt{2}$

So geometry is like



$\Rightarrow$ $T\cos 45° = \frac{m\omega^2\ell}{\sqrt{2}}$; $T\sin 45° = mg\sqrt{2}$

$1 = \frac{m\omega^2\ell}{2mg} \Rightarrow \omega^2 = \frac{2g}{\ell} \Rightarrow \omega = \sqrt{\frac{2g}{\ell}}$

6.(d)

7. (b) $\frac{V}{3} = V(1-e^{-t/RC}) \rightarrow \frac{2V}{3} = e^{-t/RC}$ ....(1)

$\frac{2V}{3} = V(1-e^{-(t+T)/RC}) \rightarrow \frac{V}{3} = e^{-(t+T)/RC}$ ....(2)

dividing equation (2) by (1)

gives $T = RC\, ln\, 2$

8.(a) Since, $T^2 = kr^3$

Differentiating the above equation

$\Rightarrow 2\frac{\Delta T}{T} = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$

Passage - 9 to 11

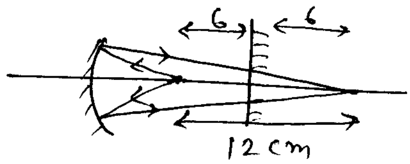
09-11 **(d, c, b)**

For concave mirror $\frac{1}{v} + \frac{1}{-12} = \frac{1}{-8}$

$\frac{1}{v} = -\frac{1}{8} + \frac{1}{12} = -\frac{3}{24} + \frac{2}{24} = -\frac{1}{24}$

$v = -24$

It has to double its volume to shift by 6 cm forward.



$(P_0, V_0, T_0) \rightarrow (P_0, 2V_0)$

$\frac{P_0V_0}{T_0} = \frac{P_0 2V_0}{T} \Rightarrow (T = 2T_0)$

$Q = nC_P\Delta T$ (isobaric $P_{ext} = P_0$)

$= \frac{P_0V_0}{T_0R}.T_0.C_P$ $= \frac{5P_0V_0}{2R}R = \frac{5P_0V_0}{2}$

Passage 12 to 16

12-14 (c, a, b)

Velocity of efflux for two holes,

$$v_1 = \sqrt{2g \times \frac{H}{2}} = \sqrt{gH} \quad ; \quad v_2 = \sqrt{2gH}$$

Thrust on tank, $F = F_1 + F_2 = \rho A v_1^2 + 2\rho A v_2^2 = 5\rho AgH$

Mass of liquids in tank = $m_1 + m_2 = 3\rho A_0 H$

$\therefore$ acceleration of tank = $\frac{5A}{3A_0}g$

$\therefore$ Correct option is (B)

Passage 15 to 16

15. (a) According to the given problem

$$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 = 5960 - 5585 - 2980 + 3645$$

$$\Delta Q = 9605 - 8565 = 1040 \text{ J}$$

$$\Delta W = W_1 + W_2 + W_3 + W_4 = 2200 - 825 - 1100 + W_4 = 275 + W_4$$

and as for cyclic process

$U_F = U_I, \qquad \Delta U = U_F - U_I = 0$

So from first law of thermodynamics, i.e., $\Delta Q = \Delta U + \Delta W$, we have

$1040 = (275 + W_4) + 0,$ i.e. $W_4 = 765$ J

16. (a) As efficiency of a cycle is defined as

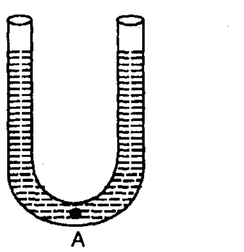
$$\eta = \frac{\text{Network}}{\text{Input heat}} = \frac{\Delta W}{(Q_1 + Q_4)} = \frac{\Delta Q}{(Q_1 + Q_4)}$$

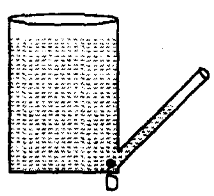
$$\eta = \frac{1040}{9605} = 0.1082 = 10.82\%.$$

Multichice Answer

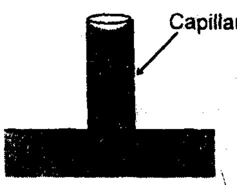
17. (a,c,d)

The pressure at any pointcan never different values. Hence (A) & (D) are not possible. (Calculate the pressure at points A & D from both their left and right)





In case of insufficient length of capillary tube the shape of meniscus is as below.



18.(a,c) According to Wien's law,

$\lambda_m T = \text{constant}$ or = constt.

So, if T is doubled, v_m is also doubled.

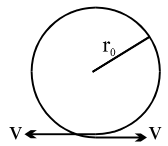
Further, according to Stefan's law.

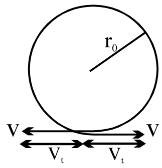
$$E \propto T^4$$

When T is doubled, E increases by a factor of 16.

19.(a,b,d) $2\pi r_0 = 2\pi r + 2Vt$

$$\therefore\ r = r_0 - \frac{Vt}{\pi} \qquad \text{(a)}$$





$Q = Bpr^2$

$$\text{emf} \therefore \frac{d\phi}{dt} = B\pi 2r\cdot\frac{dr}{dt} = B\pi 2\left(r_0 - \frac{Vt}{\pi}\right)\cdot\frac{V}{\pi} =$$

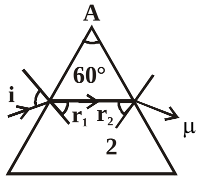
$$2BV\left(r_0 - \frac{Vt}{\pi}\right) \qquad \text{(b)}$$

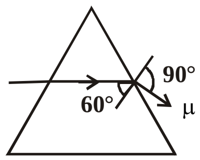
$$I = \frac{\text{emf}}{\lambda 2\pi r} = \frac{2BV\left(r_0 - \frac{Vt}{\pi}\right)}{\lambda\cdot 2\pi\left(r_0 - \frac{Vt}{\pi}\right)} = \frac{BV}{\lambda\pi} \quad \text{(d)}$$

20.(a,d) If ray corresponding to $r_{2\,max}$ pass then all r_2 will pass through prism.

r_{2max} when r_1 is min i.e., $\Rightarrow i = 0 \Rightarrow r_2 = 60°$

$2 \sin 60° = \sin 90°.\ \mu$





$\mu = 2 \text{x} \frac{\sqrt{3}}{2} = \sqrt{3}$ for all $\mu \le \sqrt{3}$ ray will pass out.

Matrix Match

21. (a-q), (b-p), (c-s), (d-r)

$q_i = 2 \times 8 = 16$

$W_{ext} = F_{m\,(plate)} \times d$

(Now work is done against the force between the plates and F is constant since charge will take time to change its value)

$= \frac{q_i^2}{2A\varepsilon_0} d = 64$ J.

$U_{cap} = \frac{q^2}{2C} = \frac{16^2}{2\times1} = 128J.$ (Here $q = q_i$ and $C = 1$)

$W_{ext} + W_{batt} = \Delta U_{cap} + heat$;

$U_{cap_{final}} = \frac{1}{2}CV^2 = \frac{1}{2}\times1\times8^2 = 32$

$U_{cap_{final}} = \frac{1}{2}CV^2 = \frac{1}{2}\times1\times8^2 = 32$

$128 - 32 = heat$
$heat = 96J$

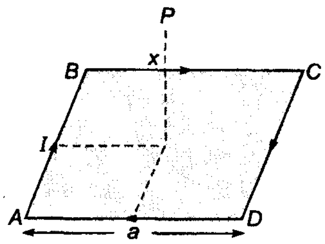
Subjective Type

22. **(0125)**

23. **(0005)**

Perpendicular diatance of P from each side of loop is,

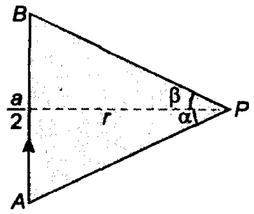
$F = \sqrt{x^2 + \frac{a^2}{}} = 4cm.$



Due to AB, magnetic field at P is given by

$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$

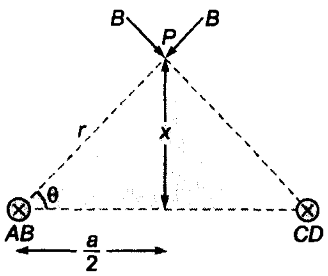
Where α and β are shown in clearly in figure.



$\sin\alpha = \sin\beta = \frac{\frac{a}{2}}{\sqrt{r^2 + \frac{a^2}{4}}} = \frac{3}{5}$

So, $B = \frac{\mu_0 I}{4\pi \times (4\,cm)} \times \frac{6}{5} = 9\times10^{-5}$ Tesla

The magnetic field at P is vector sum of magnetic field at P due to all 4 wires. Due to AB and CD we have shown the situation in adjacent figure.



Resultant magnetic field at P is ;

$B_0 = 4B\cos\theta = 2.7\times10^{-4}$ Tesla $= 0.27\,mT$

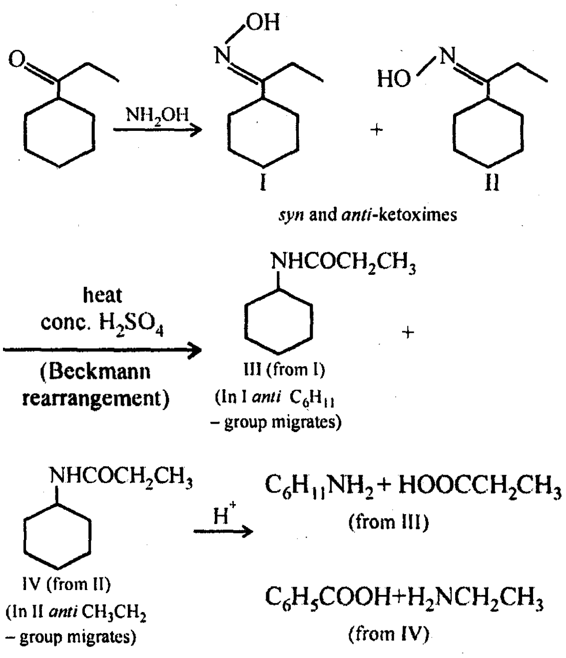
$= 0.054\times5\,mT$

So, $K = 5$

DETAILED SOLUTION

Single Choice Question

Sol.24 (d)



Sol.25 (d)

Sol. 26 (b) The molar conductivity of the dissociated form of crotonic acid is

$$\Lambda_m(HC) = \Lambda_m(HCl) + \Lambda_m(NaC) - \Lambda_m(NaCl)$$

$$= (426 + 83 - 126)\,\Omega^{-1}\,cm^2\,mol^{-1}$$

$$= 383\Omega^{-1}\,cm^2 mol^{-1}$$

The molar conductivity of HCl,

$$\Lambda_m(HC) = \frac{\kappa}{C} = \frac{3.83\times10^{-5}\Omega^{-1}\,cm^{-1}}{0.001}\times1000$$

$$= 38.3\Omega^{-1}\,cm^2\,mol^{-1}$$

The degree of dissociation,

$$\alpha = \frac{\Lambda_m(HC)}{\Lambda_m^{\infty}(HCl)} = \frac{(38.3\Omega^{-1}\,cm^2\,mol^{-1})}{(383\Omega^{-1}\,cm^2 mol^{-1})} = 0.1$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{(10^{-3})(0.1)^2}{1-0.1} = 1.11\times10^{-5}$$

Sol.27 (d) $100 = P_A^0(0.5) + P_B^0(0.5)$ ....(i)

$$0.2 = \frac{P_A^0 \times 0.5}{100}$$

$$\Rightarrow P_A^0 = \frac{20}{0.5} = 40\,\text{torr} \text{ and } P_B^0 = 160\,\text{torr}$$

Sol.28 (d) $Ni_{0.9}O$ $Ni^{+4} \rightarrow x$

$Ni^{+2} \rightarrow (0.9 - x)$

By using charge balance

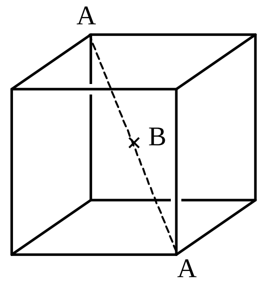
$\Rightarrow 4x + 2(0.9 - x) = 2$

$\Rightarrow 2x = 0.2$

$\Rightarrow x = 0.1$

$$\%\text{ of } Ni^{+4} = \frac{0.1}{0.9}\times100\% = 11.11\%$$

Sol.29 (c)



$$A_{4-\frac{1}{4}}B_3 = A_{\frac{15}{4}}B_3 = A_{\frac{5}{4}}B_1 = A_5B_4$$

Sol. 30(c)

Sol. 31 (d)

Sol..32 (b)

Sol.33 (d)

Sol.34 (c) $As_2S_3 + (NH_4)_2S_2 \rightarrow AsS_4^{3-}$

Sol.35 (b)

Sol.36 (c)

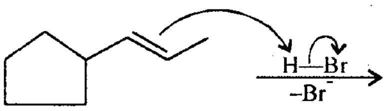
Sol.37 (a)

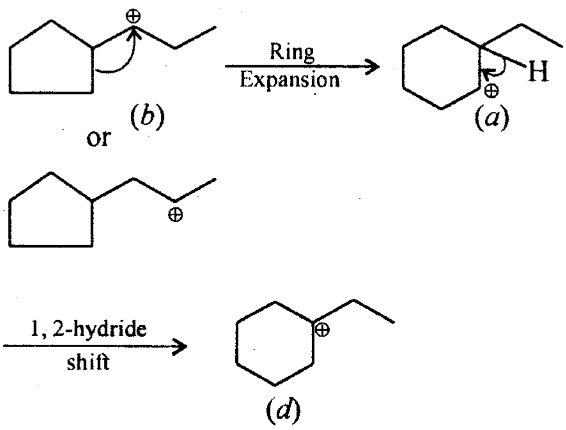
Sol.38 (b)

Sol.39 (c)

One or More than One Correct Choice Type

Sol.40 (a,b,d)





Sol.41 (a,c,d)

Sol.42 (d)

Sol. 43. (a, b, d)

Match the Column

Sol.44 A-p,s; B-p,q,r,s; C-p,r; D-p,q,s

Subjective Type

Sol.45 (6)

Sol.46 (20)

$x = 12, y = 8$

$x + y = 20$

Solution Paper - 01

DETAILED SOLUTION

Single Correct Choice Type

Sol.47(d) Put $e^x = t \Rightarrow$ t varies from $\pi/6$ to $\pi/3$

(Using King)

$$I = \int_{\pi/6}^{\pi/3} \sin^2 t\, dt \quad \text{or} \quad I = \int_{\pi/6}^{\pi/3} \cos^2 t\, dt$$

$$2I = \frac{\pi}{6} \quad \Rightarrow \quad I = \frac{\pi}{12} \text{ Ans.}$$

Sol.48(d) $T_n = 1 + 2 + 2^2 + \ldots + 2^{n-1}$

(G.P. with a = 1 and r = 2)

$= \left(2^n - 1\right)$

Hence sum = $\sum_{n=1}^{n}\left(2^n - 1\right) =$

$= [(2 + 2^2 + 2^3 + \ldots. + 2^n) - n]$

$= 2\left(2^n - 1\right) - n = 2^{n+1} - 2 - n$

$= 2^{n+1} + (0)n^2 + (-1)n + (-2)$

Hence R = 1 ; S = 0 ; T = –1 ; U = – 2

R + S + T + U = –2 Ans.

Sol.49(b) $m > 0$; $13x + 11y = 700$ ...(1)

$mx - y = 1$ ...(2)

muttiplying eq. (2) by 11)

$11\,mx - 11y = 11$ ...(3)

add (1) and (3)

$(13 + 11m)x = 711 = 3.3.79 \Rightarrow x = \frac{3.3.79}{13 + 11m}$

Hence m = 6 (for x to be integer)

$\therefore$ x = 9 and y = 53

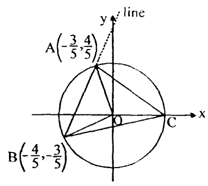
Sol.50(c) $2^y = 3^x - 1 \quad \Rightarrow \quad y\ell n2 = \ell n\left(3^x - 1\right)$

$$\ell n2\frac{dy}{dx} = \frac{3^x \cdot \ell n3}{3^x - 1} = \frac{\ell n3}{1 - 3^{-x}}$$

$$\lim_{x\to\infty}\frac{dy}{dx} = \frac{\ell n3}{\ell n2}\cdot\lim_{x\to\infty}\frac{1}{1 - 3^{-x}} = \log_2 3 \text{ Ans.}$$

Sol.51(c) Solving y = 7x + 5 and the circle $x^2 + y^2 = 1$

$A\left(-\frac{3}{5}, \frac{4}{5}\right)$ and $B\left(-\frac{4}{5}, -\frac{3}{5}\right)$



$m_{OA} = -\frac{4}{3}$; $\quad m_{OB} = \frac{3}{4}$

Hence $\angle AOD = 90° \Rightarrow \angle ACB = \frac{\pi}{4} = \tan^{-1}(1)$

Sol.52(b) **I.** Circel on (1, 3) and (3, 1) as diameter

II. $\frac{y-1}{x-3} = m$, $\frac{y-3}{x-1} = \frac{-1}{m}$, $\Rightarrow$ eliminate m, to get (b)

Sol.53(c) $a < 0;\ -\frac{b}{a} < 0 \quad \Rightarrow \quad b < 0; \frac{c}{a} < 0 \Rightarrow c > 0$

Now b – c = (–ve) – (+ ve) must be negative

bc = (–) (+ ve) must be negative

ab^2 = (–) (+ ve) must be negative

c – a = (+) – (–) must be positive

Sol.54(d) $\int_{-1}^{2} |e^x - 1|\, dx = \int_{-1}^{0}\left(1 - e^x\right)dx + \int_{0}^{2}\left(e^x - 1\right)dx$

$= x - e^x \Big|_{-1}^{0} + e^x - x \Big|_{0}^{2}$

$= -1 + 1 + \frac{1}{e} + e^2 - 2 - 1 = e^2 + e^{-1} - 3$ Ans.

Sol.55(c) $\left(\log_6 9 + \log_6 4\right) - \frac{\log_3 27}{\log_3 9} = \frac{\log_8 x}{2} - \log_8 x$

$2 - \frac{3}{2} = -\frac{1}{2}\log_8 x \quad \Rightarrow \quad \frac{1}{2} = -\frac{1}{2}\log_8 x \Rightarrow x = \frac{1}{8}$ Ans

Sol.56(d) Let $\quad S = 1^2 + 3^2 + 5^2 + \ldots. + (99)^2$

and $\quad x = 2^2 + 4^2 + 6^2 + \ldots. + (100)^2$

$\therefore\ (x - S) = \left(2^2 - 1^2\right) + \left(4^2 - 3^2\right) + \ldots. + \left(100^2 - 99^2\right)$

$= (1 + 2) + (3 + 4) + \ldots.. + (99 + 100)$

$= 1 + 2 + 3 + 4 + \ldots. + 99 + 100 = 5050$

$\therefore$ x = S + 5050 $\quad$ Ans

Sol.57(d) Note that 1/x is a cyclic function i.e. $\frac{1}{1/x} = x$.

$\therefore$ we substitute 1/x we get

$$f\left(\frac{1}{x}\right) + f(x) = \frac{1}{x}$$

Hence $x = \frac{1}{x} \Rightarrow x^2 = 1$ gives domain {–1, 1}

Sol.58(d) Family of circle

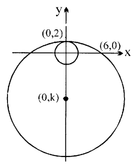
$(x – 0)^2 + (y – 2)^2 + \lambda (y – 2) = 0$

Passing through (6, 0)

$\Rightarrow \lambda = 20$

$\therefore$ circle is $x^2 + y^2 + 16y - 36 = 0$

$\Rightarrow$ centre (0, –8)



Assertion & Reason Type

Sol.59(a) Equation of director's circle is

$$(x-3)^2 + (y+4)^2 = \left(10\sqrt{2}\right)^2$$

and point (13, 6) satisfies the director circle

Sol.60(a) $AB = \sqrt{(8)^2 + (19)^2} = \sqrt{425}$;

$$AC = \sqrt{(16)^2 + (13)^2} = \sqrt{425}$$

$\therefore$ Δ is isosceles

Sol.61(b) $f(x) = \begin{cases} 1 & x = 1 \\ x & x < 1 \text{ in the immediate neighbourhood} \\ 1/x & x > 1 \text{ in the immediate neighbourhood} \end{cases}$

Hence $f(1^+) = f(1^-) \Rightarrow \lim\limits_{x \to 1} f(x)$ exists

$\therefore$ $f(1^+) = f(1^-) = f(1) = 1$

Sol.62(d) The joint equation of y = x

and y = –x is (x – y)(x + y) = 0

i.e. $x^2 - y^2 = 0$

Multiple Correct Choice Type

Sol.63(a,c,d) C_1 : (0, 0), radius 4 , C_2 : (6, 0), radius 2 circle touch externally.

Sol.64(a,c) Circles touch externally

$C_1(2,3)$; $r_1 = 5$

$C_2 = (-3,-9)$; $r_2 = 8$

Sol.65(c, d)

Sol.66 (a, c, d)

Matrix – Match Type

Sol.67 A- s ; B - r ; C - p ; D - q.

(A) $\log_{1/3}\left(\log_8 \frac{x^2 - 2x}{x-3}\right) < 0$;

$$\log_8\left(\frac{x^2-2x}{x-3}\right) > 1 \; ; \; \frac{x^2-2x}{x-3} > 8$$

$$\Rightarrow \frac{(x-6)(x-4)}{x-3} > 0$$

also $\frac{x(x-2)}{x-3} > 0 \Rightarrow x \in (0,2) \cup (3,\infty)$

$\Rightarrow x \in (3,4) \cup (6,\infty)$

$\Rightarrow a_1 + a_2 + a_3 = 13 \Rightarrow$ (s)

(B) Here a = 0 ; b = 0; c = –6

2g = –8 ; 2f = 9, 2h = k

$abc + 2fgh – af^2 – bg^2 – ch^2 = 0$

$2fgh = ch^2$

$\Rightarrow$ h = 0 $\Rightarrow$ k = 0 and

$2fg = ch \Rightarrow 9(–4) = (–6)\frac{k}{2}$

$\Rightarrow$ k = 12 or 0 $\Rightarrow$ sum = 12 $\Rightarrow$ (r)

(C) $\log\left(1+\frac{1}{1}\right) + \log\left(1+\frac{1}{2}\right) + ... + \log\left(1+\frac{1}{n}\right) = 1$

$$\log\left(2 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} .... \frac{n+1}{n}\right) = 1$$

$$\log_{10}(n+1) = 1$$

n + 1 = 10 $\Rightarrow$ n = 9 $\Rightarrow$ (p)

(D) $AM = \frac{\log 1 + \log 2 + \log 2^2 + \log 2^3 + ... + \log 2^{n-1}}{n}$

$$= \frac{\log\left(2^{1+2+3+...+(n-1)}\right)}{n} = \frac{\log 2^{\frac{(n-1)n}{2}}}{n}$$

$$AM = \frac{1}{n}\log 2^{\frac{(n-1)n}{2}} = \log 2^{\frac{(n-1)}{2}}$$

$$\therefore \log_{10} 2^{\frac{(n-1)}{2}} = \log 2^5$$

$\therefore\ 2^{\frac{(n-1)}{2}} = 2^5 \Rightarrow \frac{n-1}{2} = 5$

$\Rightarrow n = 11 \qquad \Rightarrow (q)$

SECTION - IV

Subjective Type

Sol.68[5250] $\int_0^{\pi/2} (\sin x + a\cos x)^3 dx - \frac{4a}{\pi-2}\int_0^{\pi/2} x\cos x\, dx = 2$

Let $I_1 = \int_0^{\pi/2} (\sin x + a\cos x)^3 dx$

$= \int_0^{\pi/2} \left(\sin^2 x + a^3\cos^3 x + 3a\sin^2 x\cos x + 3a^2\sin x\cos^2 x\right)dx$

$= \int_0^{\pi/2} \sin^3 x\, dx + a^3\int_0^{\pi/2}\cos^3 x\, dx + 3a\int_0^{\pi/2}\sin^2 x\cos x\, dx$

$+ 3a^2\int_0^{\pi/2}\sin x\cos^2 x\, dx$

$= \frac{2}{3} + \frac{2a^3}{3} + 3a\left.\frac{\sin^3 x}{3}\right]_0^{\pi/2} + \frac{3a^2}{3}[-\cos^3 x]\Big]_0^{\pi/2}$

$= \frac{2}{3} + \frac{2a^3}{3} + a + a^2 = \frac{2a^3}{3} + a^2 + a + \frac{2}{3}$

$I_2 = \int_0^{\pi/2} \underbrace{x}_{1}\cdot\underbrace{\cos x}_{11}\, dx = x\sin x|_0^{\pi/2} - \int_0^{\pi/2}\sin x\, dx$

$= x\sin x + \cos x|_0^{\pi/2} \qquad = \frac{\pi}{2} - 1 = \frac{\pi-2}{2}$

$\therefore\ I = \frac{2a^3}{3} + a^2 + a + \frac{2}{3} - \frac{4a}{\pi-2}.\frac{\pi-2}{2}$

or $I = \frac{2a^3}{3} + a^2 - a + \frac{2}{3} - 2a = 2$

or $2a^3 + 3a^2 - 3a + 2 = 6 \Rightarrow 2a^3 + 3a^2 - 3a - 4 = 0$

$a_1 + a_2 + a_3 = -\frac{3}{2} \Rightarrow \sum a_1a_2 = -\frac{3}{2}$

$\therefore\ \sum a_1^2 = a_1^2 + a_2^2 + a_3^2 = (a_1 + a_2 + a_3)^2$

$-2\sum a_1a_2 = \frac{9}{4} + 3 = \frac{21}{4}$

$\therefore\ 1000\sum a_1^2 = 1000\times\frac{21}{4} = 250\times 21 = 5250$

Sol.69[0006] $I = \int_{1/3}^{1}\left(x - x^3\right)^{1/3}\cdot x^{-4}\, dx$

$I = \int_{1/3}^{1} x\left(x^{-2} - 1\right)^{1/3}\cdot x^{-4}\, dx$

$= \int_{1/3}^{1}\left(x^{-2} - 1\right)^{1/3}\cdot x^{-3}\, dx$

Put $x^{-2} - 1 = t^3 \Rightarrow -2x^{-3}dx = 3t^2dt$

if $x = 1$ then $t = 0$

if $x = 1/3$ then $t = 2$

$\therefore\ I = +\frac{3}{2}\int_0^2 t - t^2\, dt = +\frac{3}{2}\cdot\left.\frac{t^4}{3}\right|_0^2$

$= +\frac{3}{8}[16] = +6$